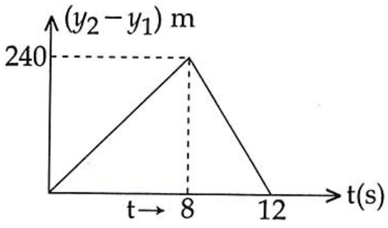
RISE

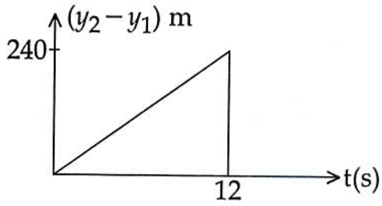
Institute with a Difference

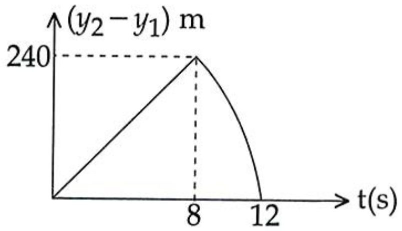
JEE Mains 2015 Solutions

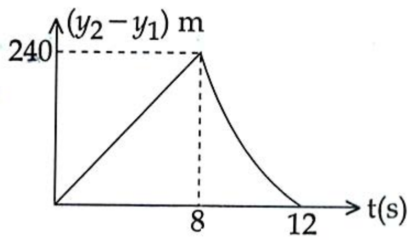
PART- A : PHYSICS

1. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first?
(Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10\ m/s^2$). (The figures are schematic and not drawn to scale)

(1) 

(2) 

(3) 

(4) 

1. (3)
For the second stone time required to reach the ground is given by

$$y = ut - \frac{1}{2}gt^2$$

$$-240 = 40\,t - \frac{1}{2} \times 10 \quad t^2$$

$$\therefore\ 5t^2 - 40\,t - 240 = 0$$

$$(t - 12)(t + 8) = 0$$

$$\therefore\ t = 12\ s$$

For the first stone :

$$-240 = 10t - \frac{1}{2} \times 10 \quad t^2$$

$$\therefore\ -240 = 10t - 5t^2$$

$$5t^2 - 10t - 240 = 0$$

$$(t - 8)(t + 6) = 0$$

$$T = 8s$$

During first 8 seconds both the stones are in air :

$$\therefore\ y_2 - y_1 = (u_2 - u_1)\,t = 30\,t$$

$\therefore$ graph of $(y_2 - y_1)$ against t is a straight line.

After 8 seconds

$$y_2 = u_2t - \frac{1}{2}gt^2 - 240$$

Stones two has acceleration with respect to stone one. Hence graph (3) is the correct description.

2. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1s resolution. The accuracy in the determination of g is

(1) 2 % (2) 3 % (3) 1 % (4) 5 %

2. (2)

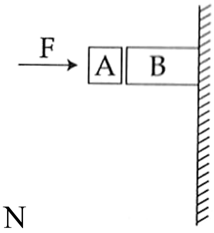
$$^{2} = 4\pi \cdot \frac{L}{g}$$

$$\therefore g = 4\pi^2 \frac{L}{T^2}$$

$$\frac{\Delta g}{g} \times 100 = \frac{\Delta L}{L} \times 100 + 2 \cdot \frac{\Delta T}{T} \times 100$$

$$= \frac{0.1}{20} \times 100 + 2 \times \frac{1}{90} \times 100 = 0.5 + 2.2 = 2.7 \simeq 3.$$

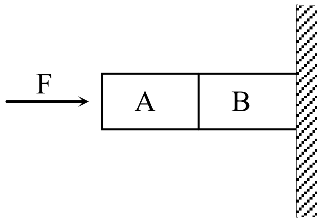
3. Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is :



(1) 100 N (2) 80 N (3) 120 N (4) 150 N

3. (3)

For Block A :



$m_1 g = \mu_1 F$

$20 = 0.1 \times F$

$F = \frac{20}{0.1} = 200$ N

Frictional force on block A in upward direction = $\mu_1 F = 0.1 \times 200 = 20$ N

Block A exert a frictional force of 20 N on block B in down–ward direction.

$\therefore$ For block B :

$\therefore \quad \mu_2 F = m_2 g + \mu_1 F = 100 + 20 = 120$ N

4. A particle of mass m moving in the x direction with speed 2v is hit by another particle of mass 2m moving in the y direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to :

(1) 44 % (2) 50 % (3) 56 % (4) 62 %

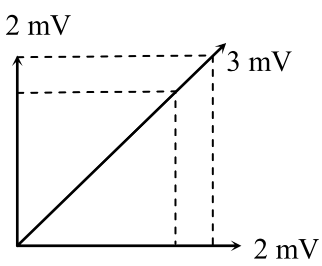
4. (3)

$E_1 = \frac{1}{2} m(2v)^2 + - 2m . v$

$= \frac{1}{2} m . 4v^2 + mv$

$= 2mv^2 + mv^2$

$= 3\ mv^2$



After Collision

$3mV = \sqrt{2} . 2mv$

$V = \frac{2\sqrt{2}\ v}{3}$

$$E_2 = \frac{1}{2}3m.\left(\frac{2\sqrt{2}\,v}{}\right)^2 = \frac{3}{2}m.\frac{8v^2}{} = \frac{4}{3}v^2$$

$$E_1 - E_2 = 3v^2 - \frac{4}{3}v = \frac{9v^2 - 4v}{3} = \frac{5}{3}v^2$$

$$\therefore \frac{E_1 - E}{E_1} = \frac{\frac{5}{3}v^2}{3v^2} = \frac{5}{9}$$

Percentage loss = $\frac{5}{9} \times 100 = 55.6 \approx 56.$

5. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h the z_0 is equal to :

(1) $\frac{h^2}{4R}$ (2) $\frac{3h}{4}$ (3) $\frac{5h}{8}$ (4) $\frac{3h^2}{8R}$

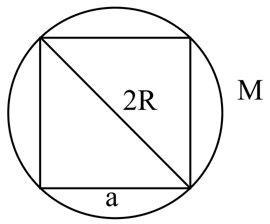
5. (2)

6. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is :

(A) $\frac{MR^2}{32\sqrt{2}\,\pi}$ (2) $\frac{MR^2}{16\sqrt{2}\,\pi}$ (3) $\frac{4MR^2}{9\sqrt{3}\,\pi}$ (4) $\frac{4MR^2}{3\sqrt{3}\,\pi}$

6. (3)

Figure alongside shows a solid sphere of mass M.
The radius of the sphere is R.
The volume of the sphere is



$$V = \frac{4\pi}{3}\ ^3$$

The density of the sphere is $\rho = \frac{M}{V} = \frac{M}{\frac{4\pi}{3}R^3} = \frac{3M}{4\pi R^3}$

From this solid sphere a cube of maximum possible volume is cut.

Therefore $2R = \sqrt{3}$ a, where a is the length of the side of the cube of maximum volume

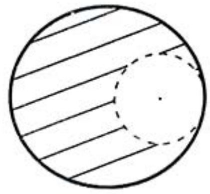
$$a = \frac{2R}{\sqrt{3}}$$

Mass of the cube is $M' = \rho\ ^3 = \frac{3M}{\pi R^3}\frac{8R^3}{3\sqrt{}} = \frac{2M}{\sqrt{3}\pi}$

The moment of inertia of the cube is

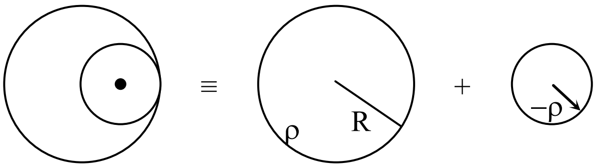
$$M'\frac{a^2}{6} = \frac{2M}{\sqrt{3}\,\pi} \times \frac{1}{} \times \left(\frac{2R}{\sqrt{}}\right)^2 = \frac{8MR^2}{18\sqrt{3}\,\pi} \quad \frac{4MR}{9\sqrt{3}\,\pi}$$

7. From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavity thus formed is :
(G = gravitational constant)



(1) $\frac{-GM}{2R}$ (2) $\frac{-GM}{R}$ (3) $\frac{-2GM}{3R}$ (4) $\frac{-2GM}{R}$

7. (2)



Potential at internal point of solid sphere at a distance ‘r’

$$V = -\frac{GM}{R}\left[\frac{3}{2}-\frac{r^2}{2R^2}\right]$$

at $\quad r = \frac{R}{2}$

$$V_1 = -\frac{GM}{R}\left[\frac{3}{2}-\frac{R^2}{8R^2}\right] = -\frac{11}{8}\frac{GM}{R}$$

Because of sphere removed.

$$V_2 = \frac{3}{2}\cdot\frac{G\frac{M}{8}}{\frac{R}{2}} = \frac{3}{8}\frac{GM}{R}$$

Net potential, $V = V_1 + V_2 = -\frac{11}{8}\frac{GM}{R}+\frac{3}{8}\frac{GM}{R} = -\frac{GM}{R}$

8. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to :

(g = gravitational acceleration)

(1) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$ (2) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{}$

(3) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (3) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{}{Mg}$

8. (1)

$T = 2\pi\sqrt{\frac{\ell_1}{g}},\ T_M = 2\ \sqrt{\frac{\ell}{}}$

$T^2 = 4\pi^2\cdot\frac{\ell_1}{g},\quad T_M^2 = 4\pi^2\cdot\frac{\ell}{}$

$\frac{T^2}{T^2} = \frac{\ell_2}{\ell_1}x$

$\frac{T_M^2}{T^2}-1 = \frac{\ell_2}{\ell_1}- \ = \frac{\ell_2-\ell_1}{\ell}$

$\gamma = \frac{Mg}{A}\cdot\frac{\ell_1}{(\ell_2-\ell\ )}$

$\therefore\ \frac{1}{\gamma} = \frac{A}{Mg}\cdot\frac{(\ell_2-\ell\ )}{\ell_1} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2-1\right]$

9. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $P = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is :

(1) $T \propto e^{-R}$ (2) $T \propto e^{-3R}$ (3) $T \propto \frac{1}{R}$ (4) $T \propto \frac{1}{R^3}$

9. (3)

$$P = -\left(\frac{U}{V}\right) \propto T^4$$

For an ideal gas $PV = nR'T$

$$\therefore P = \frac{nR'T}{V} \qquad (R' - \text{molar gas constant})$$

$$\therefore \frac{nR'T}{V} \propto T^4$$

$$\frac{nR'}{V} \propto T^3$$

$$\therefore \frac{1}{V} \propto T^3$$

$$\frac{1}{\frac{4\pi}{3}R^3} \propto T^3$$

$$T^3 \propto \frac{1}{R^3}$$

$$T \propto \frac{1}{R}$$

10. A solid of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways :

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is :

(1) $\ell n2,\ 4\ell n2$ (2) $\ell n2,\ \ell n2$ (3) $\ell n2,\ 2\ell n2$ (4) $2\ell n2,\ 8\ell n2$

10. (No option matches)

Entropy is a state function so depends on initial temperature and final temperature.

$$\text{Case – I : } \quad \Delta s = c\left[\int_{373}^{423}\frac{dT}{T} + \int_{423}^{423}\frac{dT}{T}\right] \ \ell\left(\frac{473}{373}\right)$$

$$\text{Case – II : } \quad \Delta s = c\left[\int_{373}^{385.5}\frac{dT}{T} + \int_{385.5}^{398}\frac{dT}{T} + \ldots\ldots + \int_{460.5}^{473}\frac{dT}{T}\right] = \ell n\left(\frac{473}{373}\right)$$

Note : In the question temperatures are given in °C, but should be taken in kelvin so no option matches.

11. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is :

$$\left.\gamma = \frac{C_p}{C_v}\right)$$

(1) $\frac{3\gamma +}{6}$ (2) $\frac{3\gamma -}{6}$ (3) $\frac{\gamma + 1}{2}$ (4) $\frac{\gamma - 1}{2}$

11. (3)

Average time taken collisions

$t = \frac{\lambda}{v_{runs}}$ λ = mean free path

$$\lambda = \frac{1}{\sqrt{2}\,\frac{N}{V}\pi\ \ ^2}$$

N = Number of molecular

V = volumes

$$\therefore\ t = \frac{\dfrac{1}{\sqrt{2}\,\frac{N}{V}\pi\ \ ^2}}{\sqrt{\dfrac{3RT}{M}}} = \frac{\sqrt{M}\cdot V}{\sqrt{2}\,N\pi d^2\,\sqrt{3R}\,\sqrt{T}}$$

$$t = k\frac{V}{\sqrt{T}}$$

where k is a constant $= \frac{\sqrt{M}}{\sqrt{2}\,N\pi d^2\sqrt{3R}}$

$$\therefore\quad T \propto \frac{V^2}{t^2}$$

For adiabatic process

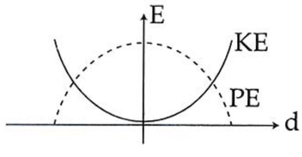
$TV^{\gamma-1}$ = constant

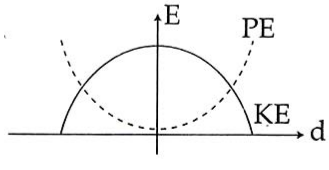
$$\frac{V^2}{t^2}V^{\gamma-1} = \text{constant}$$

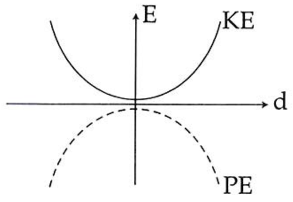
$$\frac{V^{\gamma+1}}{t^2} = \text{constant}$$

$\therefore\ t \propto V^{\frac{\gamma+1}{2}}$ $\therefore\ q = \frac{\gamma+1}{2}$

12. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly?
(graph are schematic and not drawn to scale)

(1)


(2)


(3)


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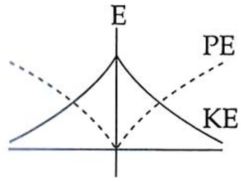


12. (2)
A simple pendulum performs simple harmonic motion.
For simple harmonic motion.

Potential energy goes as $\frac{1}{2}m\omega\ x^2$

The maximum potential energy being $\frac{1}{2}m\omega\ A^2$.

Kinetic energy goes as $\frac{1}{2}m\ \ ^2\left(A^2 - x^2\right)$. Graph (2) fits these plots.

13. A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 ms^{-1}) close to :

(1) 6 % (2) 12 % (3) 18 % (4) 24 %

13. (2)
A train is moving on a straight track with speed vs = 20 m/s. It is blowing its whistle at the frequency of ν_0 = 1000 Hz.
A person is standing near the track.
As the train approaches him, the frequency of the whistle as heard by him is

$$\nu_1 = \nu_0 \frac{v}{v - v_s} \quad \text{(where v = 320 m/s is the speed of sound in air)}$$

$$\therefore\ \nu_1 - 1000 \times \frac{320}{(320-20)} = 1000 \times \frac{320}{300} = 1066.67 \text{ Hz.}$$

As the train goes away from him, the frequency of the whistle as heard by him is

$$\nu_2 = \nu_0 \frac{v}{v + v_s}$$

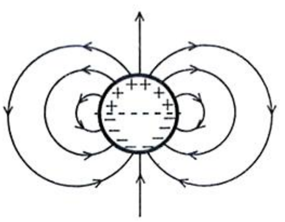
$$\nu_2 = 1000 \times \frac{320}{(320+20)} = 1000 \times \frac{320}{340} = 941.17 \text{ Hz}$$

Percentage change in frequency heard by the person standing near the track as the train passes him is

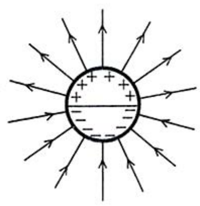
$$\frac{\nu_1 - \nu}{\nu_0} \times 100 = \frac{1066.67 - 941.17}{1000} \times 100 = \frac{125.5}{10} = 12.55\% \approx 12\%$$

14. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge −σ in the lower half. The electric field lines around the cylinder will look like figure given in : (figure are schematic and not drawn to scale)

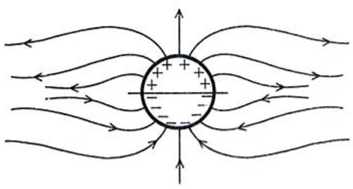
(1)



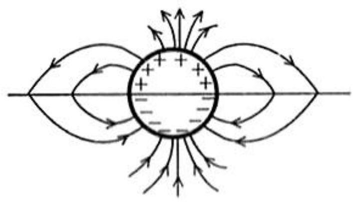
(2)



(3)



(4)

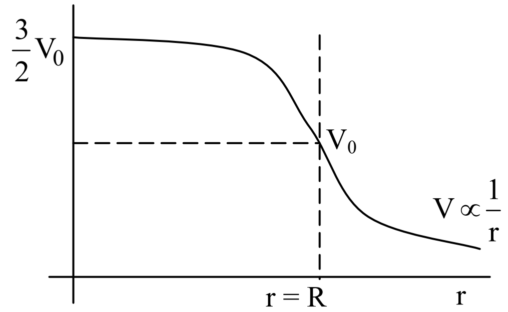


14. (1)
Nature of electric field lines.

15. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}$, $\frac{5V_0}{4}$, $\frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then

(1) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
(2) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$
(3) $R_1 = 0$ and $R_2 < (R_4 - R_3)$
(4) $2R < R_4$

15. (3), (4)
Variation of potential of solid sphere



Potential insider is given by

$$V_{in} = \frac{Q}{4\pi\ _0R}\left[\frac{3}{2}\ \ \frac{r^2}{2R^2}\right]$$

$$V_s = \frac{Q}{4\pi\ _0} = \qquad \text{(given)}$$

$$V_{out} = \frac{Q}{4\pi\ _0}$$

$$V_C = \frac{3}{2}$$

$\frac{5}{4}V_o$ is possible inside the sphere

$$\therefore\ \frac{5}{4}V_o = V\left(\frac{3}{2} - \frac{r^2}{2R^2}\ \ \Big|\ r = \frac{R}{2}\right)$$

$$r = \frac{R}{\sqrt{2}} = \quad _2$$

$\frac{3V_o}{4}$ (possible outside)

$$\frac{3V_o}{4} = \frac{Q}{4\pi\ _0 r}$$

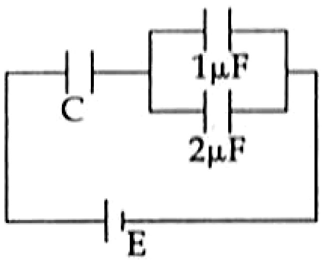
$$\frac{3}{4}\frac{Q}{4\pi\in_0} = \frac{Q}{4\pi\in\ r}$$

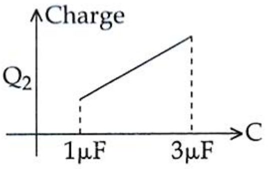
$$r = \frac{4}{3}R\ (=R_3)$$

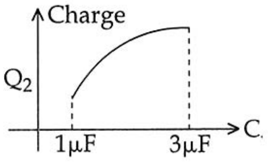
Similarly
$r = 4\ R\ = R_4$

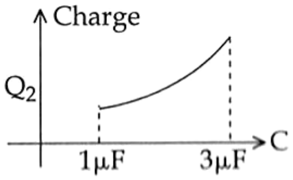
So $\frac{3}{2}V_o$ possible at $R_1 = 0$. We can see that $R_2 <\ (R_4 - R_3)$

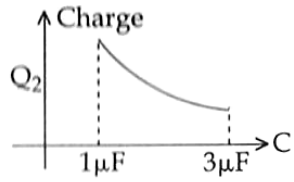
16. In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1μF to 3μF. Q_2 as a function of 'C' is given properly by : (figures are drawn schematically and are not to scale)



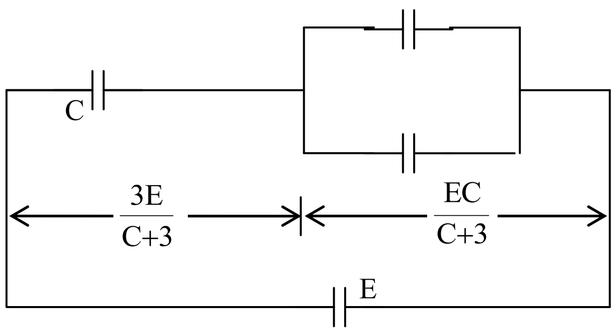
(1) 

(2) 

(3) 

(4) 

16. (2)



$\therefore\ Q_2 = \frac{2EC}{C+3}$

$Q_2 = \frac{2E}{1+\frac{3}{C}}$

C increases from 1 to 3 μF, so Q_2 increases as E is constant. Checking the concavity of the function using 2nd derivative we find option (2) is correct.

17. When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is $2.5 \times 10^{-4}\ ms^{-1}$. If the electron density in the wire is $8 \times 10^{28}\ m^{-3}$, the resistivity of the material is close to :

(1) $1.6 \times 10^{-8}\ \Omega m$ (2) $1.6 \times 10^{-7}\ \Omega m$ (3) $1.6 \times 10^{-6}\ \Omega m$ (4) $1.6 \times 10^{-5}\ \Omega m$

17. (4)

$\upsilon_d \quad \frac{i}{A\,n\,e} \qquad \therefore\ \frac{i}{A} = \upsilon_d . n\,e$

$i = \frac{}{R} = \frac{V}{\rho \frac{\ell}{A}}$

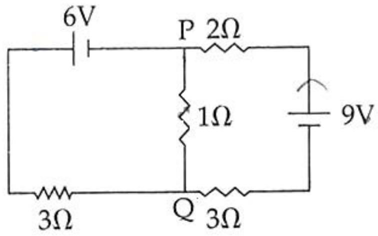
$\therefore\ \rho = \frac{V}{\frac{i}{A}\ell}$

$\rho = \frac{V}{\upsilon_d \cdot n\,e\,\ell}$

$\rho = \frac{5}{2.5\times10^{-4}\times8\times10^{28}\times1.6\times10^{\ 19}\times0.1} = \frac{2}{8\times1.6\ \ 10^{4}} = \frac{1}{6.4}\ 10^{-4} = 1.6\times10^{-5}\ \Omega - m$

18. In the circuit shown, the current in the 1Ω resistor is :

(1) 1.3 A, from P to Q
(2) 0 A
(3) 0.13 A, from Q to P
(4) 0.13 A, from P to Q



18. (3)

Apply KVL,

$$-6 + I_1 + 3I = 0$$

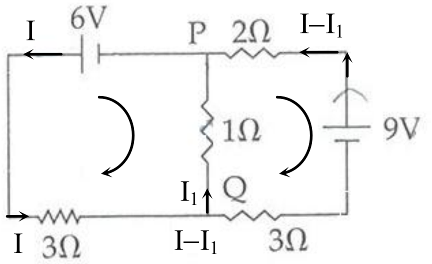
$\therefore \quad 3I + I_1 = 6 \quad \ldots(1)$

and $\quad 2(I - I_1) - 9 + 3(I - I_1) - I_1 = 0$

$\therefore \quad 5I - 6I_1 = 9 \quad \ldots(2)$

Solving equations (1) and (2)

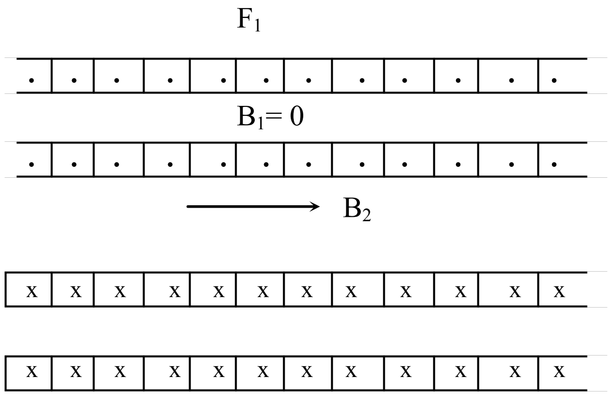
$$I_1 = \frac{3}{23} = 0.13\text{ A} \qquad \text{Q to P}$$



19. Two coaxial solenoids of different radii carry current I in the same direction. Let $\vec{F_1}$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F_2}$ be the magnetic force on the outer solenoid due to the inner one. Then :

(1) $\vec{F_1} = \vec{F_2} = 0$
(2) $\vec{F_1}$ is radially inwards and $\vec{F_2}$ is radially outwards
(3) $\vec{F_1}$ is radially inwards and $\vec{F_2} = 0$
(4) $\vec{F_1}$ is radially outwards and $\vec{F_2} = 0$

19. (1)



Solenoid consists of circular current loops which are placed in uniform field, so magnetic fore on both solenoids = 0.

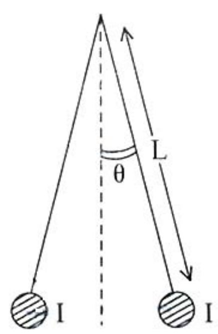
$F_1 = F_2 = 0$

20. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle 'θ' with the vertical. If wires have mass λ per unit length then the value of I is :

(g = gravitational acceleration)

(1) $\sin\theta\sqrt{\dfrac{\pi\lambda gL}{\mu_0\cos\theta}}$ (2) $2\sin\theta\sqrt{\dfrac{\pi\lambda gL}{\mu_0\cos\theta}}$

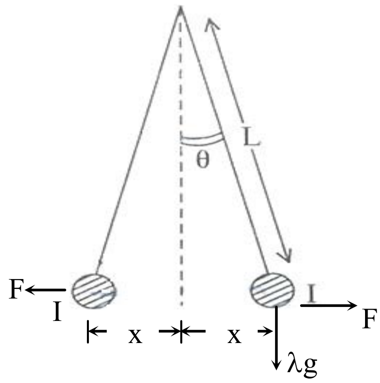
(3) $2\sqrt{\dfrac{\pi gL}{\mu_0}\tan\theta}$ (4) $\sqrt{\dfrac{\pi\lambda gL}{\mu_0}\tan\theta}$



20. (2)

$$F = \frac{\mu_0 I^2}{2\pi . 2x} \qquad x = \ell \sin\theta$$

$$F = \frac{\mu_0 I^2}{4\pi\ell \sin\theta}$$



For equations

$$T\sin\theta \quad \frac{\mu_0 I^2}{4\pi\ell\sin\theta}$$

and $\quad T\cos\theta = \lambda g$

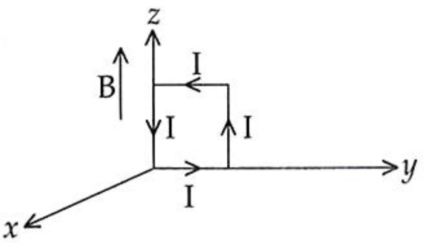
dividing $\tan\theta = \dfrac{\mu_0 I^2}{4\pi\ell\sin\theta\,\lambda g}$

$\therefore \qquad I^2 = \dfrac{4\pi\ell\sin^2\theta\lambda g}{\mu_0 \cos}$

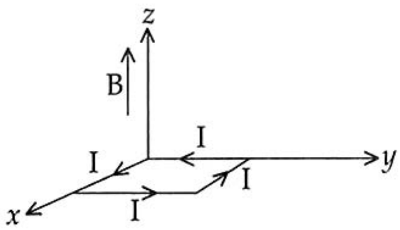
$$I = 2\sin\sqrt{\frac{\pi\ell\lambda g}{\cos\theta}}$$

21. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below :

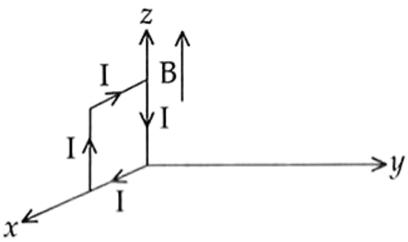
(a)



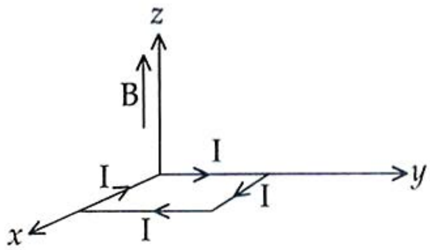
(b)



(c)



(d)



If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?

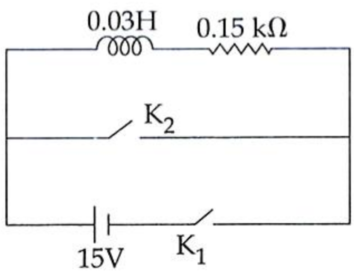
(1) (a) and (b), respectively (2) (a) and (c), respectively
(3) (b) and (d), respectively (4) (b) and (c), respectively

21. (3)

Because in both option (b) and (d)

$F_{net} = 0 \;\; \tau_{net} = 0$

22. An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at t = 0, K_1 is opened and key K_2 is closed simultaneously. At t = 1 ms, the current in the circuit will be :($e^5 \cong 150$)

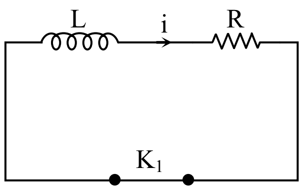


(1) 100 mA (2) 67 mA
(3) 6.7 mA (4) 0.67 mA

22. (4)

When switch K_1 closed for long time current through inductor

$$I = \frac{E}{R} = \frac{15}{0.15\times10^3} = 0.1\,A$$



when K_1 opened and K_2 is closed

$$i = I e^{\,t/\tau}$$

$$t = 1\text{ ms} = 1 \times 10^{-3}\text{ s}$$

$$\tau = \frac{L}{R} = \frac{0.03}{0.15\times10^3} = \frac{10^{-3}}{}$$

$$\therefore\ i = 0.1\,e^{\frac{-1\times10^{-3}}{\frac{10^{-3}}{5}}}$$

$$= 0.1\ e^{-5}$$

$$= \frac{0.1}{e^5} = \frac{0.1}{150} = \frac{1}{15}\times10^{-2}\times10\ \text{ mA}$$

$$i = 0.67\text{ mA}$$

23. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is :

(1) 1.73 V/m (2) 2.45 V/m (3) 5.48 V/m (4) 7.75 V/m

23. (2)

$$I = \frac{P}{4\pi r^2}$$

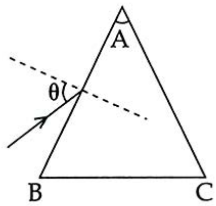
$$\therefore\ \frac{1}{2}\in_0 \ ^2 C = \frac{}{4\pi r^2}$$

$$E_0 = \sqrt{\frac{2P}{\in_0 \cdot C\cdot 4\pi r^2}}$$

Putting the values

$$E_0 = \sqrt{\frac{2\times0.1}{8.85\times10^{-12}\times3\times10^8\times4\pi\times1^2}} = 2.45\text{ V/m}$$

24. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided :



(1) $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$ (2) $\theta < \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

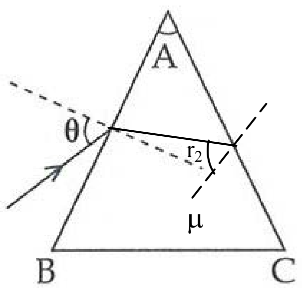
(3) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$ (4) $\theta < \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

24. (1)

$1 . \sin\theta = \mu \sin r_1$

$\mu \sin \theta_C = 1 . \sin 90$

$$\sin \theta_C = \frac{1}{\mu}$$



$$r_2 < \theta$$

$$\sin r_2 < \sin\theta$$

$$r_1 + r = A$$

$$r_1 = A - r$$

$$r_1 = A - \theta_C$$

$\sin r_1 > \sin(A - \theta_C)$
$\mu \sin r_1 > \mu \sin(A - \theta_C)$
$\sin \theta > \mu \sin(A - \theta_C)$

$$\theta > \sin^{-1}\left\{\mu \sin\left(A - \sin \frac{1}{\mu}\right)\right\}$$

25. On a hot summer night, the refractive index of air is smallest near the ground and incrases with height from the ground. When a light beam is directed horizontally, the Hyygens' principle leads us to conclude that as it travels, the light beam :
(1) becomes narrower
(2) goes horizontally without any deflection
(3) bends downwards
(4) bends upwards

25. (4)
As light beam goes from rarer medium to denser medium.

26. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is :
(1) 1 μm (2) 30 μm (3) 100 μm (4) 300 μm

26. (2)
Assuming human pupil to have a radius r = 0.25 cm.
or diameter d = 2r = 2 × 0.25 = 0.5 cm, the wavelength of light is $\lambda = 500 \text{ nm} = 5 \times 10^{-7}$ m.
We have the formula

$$\sin\theta \quad \frac{1.22\lambda}{d}$$

$$\therefore \sin\theta = \frac{1.22 \times 5 \times 10^{-7}}{0.5 \quad 10^{-2}} = \frac{1.22 \times 5 \times 10}{5 \times 10} = 1.22 \times 10^{-4}$$

The distance of comfortable viewing is D = 25 cm = 0.25 m.

Let x be the minimum separation between two objects that the human eye can resolve.

$$\therefore \sin\theta \quad \frac{x}{D} \qquad \therefore x = D \sin\theta = 0.25 \times 1.22 \times 10^{-4} = 3 \times 10^{-5} \text{ m} = 30 \text{ μm}.$$

27. As an electron makes a transition from an excited state to the ground state of a hydrogen – like atom/ion :
(1) its kinetic energy increases but potential energy and total energy decrease
(2) kinetic energy, potential energy and total energy decrease
(3) kinetic energy decreases, potential energy increases but total energy remains same
(4) kinetic energy and total energy decrease but potential energy increases

27. (1)
For an electron in the hydrogen atom we have

$$\frac{mv^2}{} = \frac{1}{4\pi \ _0} \frac{e}{r^2}$$

The centripetal force is provided by the electrostatic force of attraction.

$$\therefore mv^2 = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r}$$

$$\frac{1}{2} mv^2 = \frac{1}{8\pi \ _0} \frac{e^2}{r}$$

The potential energy of the electron is given by $-\dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{r}$

The total energy is given by

$$\frac{1}{2}mv^2-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}=\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r}-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}=-\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r}$$

As the electron makes a transition from an excited state to the ground state of a hydrogen-like atom.

r decreases therefore kinetic energy increases as K.E. = $\dfrac{1}{8\pi\varepsilon_0}\dfrac{e^2}{r}$ K.E. $\propto \dfrac{1}{r}$

r decreases therefore potential energy decreases as P.E. = $-\dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{r}$ P.E. $\propto -\dfrac{1}{r}$

r decrease therefore total energy decreases as total = $-\dfrac{1}{8\pi\varepsilon_0}\dfrac{e^2}{r}$ T.E. $\propto -\dfrac{1}{r}$

28. Match **List – I** (Fundamental Experiment) with **List – II** (its conclusion) and select the correct option from the choices given below the list :

	List – I		**List – II**
(A)	Franck-Hertz Experiment.	(i)	Particle nature of light
(B)	Photo-electric experiment.	(ii)	Discrete energy levels of atom
(C)	Davison-Germer Experiment	(iii)	Wave nature of electron
		(iv)	Structure of atom

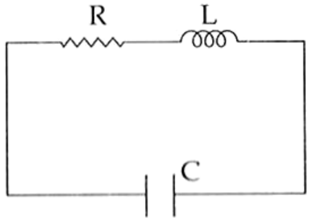
(1) (A) – (i) (B) – (iv) (C) – (iii)
(2) (A) – (ii) (B) – (iv) (C) – (iii)
(3) (A) – (ii) (B) – (i) (C) – (iii)
(4) (A) – (iv) (B) – (iii) (C) – (ii)

28. (3)
Frank–hertz experiment → Discrete energy levels of atom.
Photo-electric experiment → Particle nature of light
Davisson-Germer Experiment → Wave nature of electron.

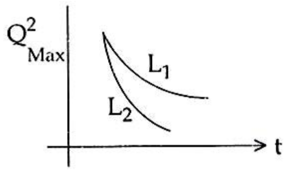
29. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequencies of the resultant signal is/are :
(1) 2 MHz only
(2) 2005 kHz, and 1995 kHz
(3) 2005 kHz, 2000 kHz and 1995 kHz
(4) 2000 kHz and 1995 kHz

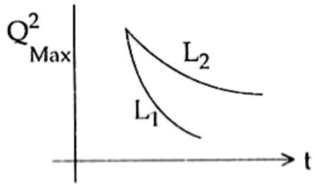
29. (3)
A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz.
The frequencies of the resultant signal can be therefore 2005 Hz, 2000 Hz, 1995 Hz.

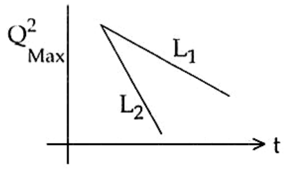
30. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below :

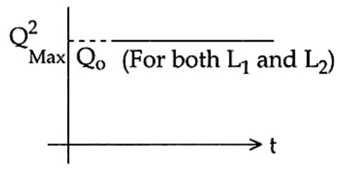


If a student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly? (plots are schematic and not drawn to scale)

(1) 

(2) 

(3) 

(4) 

30. (1)

For a damped pendulum $A = A_0 e^{-bt/2m}$

For the given circuit. $\therefore\ A = A_0 e^{-\left(\frac{R}{2L}\right)t}$

L plays the same role as m.

$\because$ $L_1 > L_2$ so with L_1 in the circuit energy dissipation in R will be slower compared to L_2 present in the circuit.

PART- B : CHEMISTRY

31. The molecular formula of a commercial resin used for exchanging ions in water softening is $C_8H_7SO_3Na$ (Mol. Wt. 206). What would be the maximum uptake of Ca^{2+} ions by the resin when expressed in mole per gram resin?

(1) $\frac{1}{103}$ (2) $\frac{1}{206}$ (3) $\frac{2}{309}$ (4) $\frac{1}{412}$

31. (4)

2 mole of $C_8H_7SO_3Na$ = 1 mole of Ca^{+2}

2×206 g $C_8H_7SO_3Na$ = 1 mole of Ca^{+2}

1 g $C_8H_7SO_3Na$ = $\frac{1}{412}$

32. Sodium metal crystallizes in a body centred cubic lattice with a unit cell edge of 4.29 Å. The radius of sodium atom is approximately :

(1) 1.86 Å (2) 3.22 Å (3) 5.72 Å (4) 0.93 Å

32. (1)

For BCC unit cell, $\sqrt{3}\,a = 4r$

$r = \frac{\sqrt{3}}{4} a = \frac{\sqrt{\ }}{\ } \times 4.29\ \text{Å} = 1.85$

33. Which of the following is the energy of a possible excited state of hydrogen?

(1) +13.6 eV (2) −6.8 eV (3) −3.4 eV (4) +6.8 eV

33. (3)

Energy in 1st excited state = −3.4 eV.

34. The intermolecular interaction that is dependent on the inverse cube of distance between the molecules is :

(1) ion–ion interaction
(2) ion–dipole interaction
(3) London force
(4) hydrogen bond

34. (2)

35. The following reaction is performed at 298 K.

$2NO(g) + O_2(g) \rightleftharpoons 2NO_2(g)$

The standard free energy of formation of NO(g) is 86.6 kJ/mol at 298 K. What is the standard free energy of formation of $NO_2(g)$ at 298 K? ($K_p = 1.6 \times 10^{12}$)

(1) $R(298)\ \ell n\,(1.6 \times 10^{12}) - 86600$
(2) $86600 + R(298)\ \ell n\,(1.6 \times 10^{12})$
(3) $86600 - \dfrac{\ell n(1.6\times10^{12})}{R(298)}$
(4) $0.5\,[2 \times 86{,}600 - R(298)\ \ell n\,(1.6 \times 10^{12})]$

35. (4)

$$\Delta G_{Rx^n} = 2\,\Delta G_{NO_2} - 2\Delta G_{NO}$$

$$\Delta G_{NO_2} = \Delta G_{NO} + \frac{1}{2}\Delta\ \ _{Rx}$$

$$= \Delta G_{NO} + \frac{1}{2}\left(-RT\ ln_e\ K_p\right)$$

$$= 0.5\left[2\times86{,}600 - R\ (298)\ ln_e(1.6\times10^{12})\right]$$

36. The vapour pressure of acetone at 20°C is 185 torr. When 1.2 g of a non–volatile substance was dissolved in 100 g of acetone at 20°C, its vapour pressure was 183 torr. The molar mass (g mol^{-1}) of the substance is :

(1) 32
(2) 64
(3) 128
(4) 488

36. (2)

$$\frac{\ ^0 - P}{P^0} = X_{solute} = \frac{n_1}{n\ \ + n_2}$$

$$\frac{185-183}{185} = \frac{2}{185} = \frac{1.2/M}{\frac{1.2}{M}+\frac{100}{58}} \Rightarrow M = 64$$

37. The standard Gibbs energy change at 300 K for the reaction $2A \rightleftharpoons B + C$ is 2494.2 J. At a given time, the composition of the reaction mixture is $[A] = \frac{1}{2}$, $[B] = 2$ and $[C] = \frac{1}{2}$. The reaction proceeds in the :

[R = 8.314 J/K/mol, e = 2.718]

(1) forward direction because $Q > K_c$
(2) reverse direction because $Q > K_c$
(3) forward direction because $Q < K_c$
(4) reverse direction because $Q < K_c$

37. (2)

$$\Delta G = -RT\ \ ln_e\,K_c$$

$$2494.2 = 8.314 \times 300\ ln_e\,K_c$$

$$K_c = e^{-1}$$

$K_c = e^{-1} = \frac{1}{2.718} = 0.36$

$Q = \frac{(B)(C)}{[A]^2} = \frac{2 \times \frac{1}{2}}{\left[\frac{1}{2}\right]^2} = 4$

$Q > K_c$, i.e. backward reaction.

38. Two Faraday of electricity is passed through a solution of $CuSO_4$. The mass of copper deposited at the cathode is :
(at. mass of Cu = 63.5 amu)
(1) 0 g (2) 63.5 g (3) 2 g (4) 127 g

38. (2)

$Cu^{+2} + 2e \longrightarrow Cu(s)$

2 mol 1 mol = 63.5

39. Higher order (>3) reactions are rare due to :
(1) low probability of simultaneous collision of all the reacting species
(2) increase in entropy and activation energy as more molecules are involved
(3) shifting of equilibrium towards reactants due to elastic collisions
(4) loss of active species on collision

39. (1)

40. 3 g of activated charcoal was added to 50 mL of acetic acid solution (0.06 N) in a flask. After an hour it was filtered and the strength of the filtrate was found to be 0.042 N. The amount of acetic acid adsorbed (per gram of charcoal) is :
(1) 18 mg (2) 36 mg (3) 42 mg (4) 54 mg

40. (1)

Amount of acetic acid adsorbed $= \frac{(0.06 - 0.042) \times 50 \times 10^{-3} \times 60}{3} = 18 \times 10^{-3} = 18$ mg.

41. The ionic radii (in Å) of N^{3-}, O^{2-} and F^- are respectively :
(1) 1.36, 1.40 and 1.71 (2) 1.36, 1.71 and 1.40
(3) 1.71, 1.40 and 1.36 (4) 1.71, 1.36 and 1.40

41. (3)

Ionic Radii order : $N^{3-} > O^{2-} > F^-$

42. In the context of the Hall–Heroult process for the extraction of $A\ell$, which of the following statements is **false**?
(1) CO and CO_2 are produced in this process
(2) $A\ell_2O_3$ is mixed with CaF_2 which lowers the melting point of the mixture and brings conductivity
(3) $A\ell^{3+}$ is reduced at the cathode to form $A\ell$
(4) $Na_3A\ell F_6$ serves as the electrolyte

42. (4)

43. From the following statements regarding H_2O_2, choose the **incorrect** statement :
(1) It can act only as an oxidizing agent
(2) It decomposes on exposure to light
(3) It has to be stored in plastic or wax lined glass bottles in dark
(4) It has to be kept away from dust

43. (1)
It can acts as an oxidising as well as reducing agent.

44. Which one of the following alkaline earth metal sulphates has its hydration enthalpy greater than its lattice enthalpy?
(1) $CaSO_4$ (2) $BeSO_4$ (3) $BaSO_4$ (4) $SrSO_4$

44. (2)
$BaSO_4$ is least soluble.
$BeSO_4$ is most soluble.

45. Which among the following is the most reactive?
(1) Cl_2 (2) Br_2 (3) I_2 (4) ICl

45. (4)
The interhalogen compounds are generally more reactive than halogens (except F_2).

46. Match the catalysts to the correct processes :

Catalyst			**Process**
(A)	$TiCl_3$	(i)	Wacker process
(B)	$PdCl_2$	(ii)	Ziegler – Natta polymerization
(C)	$CuCl_2$	(iii)	Contact process
(D)	V_2O_5	(iv)	Deacon's process

(1) (A) – (iii), (B) – (ii), (C) – (iv), (D) – (i)
(2) (A) – (ii), (B) – (i), (C) – (iv), (D) – (iii)
(3) (A) – (ii), (B) – (iii), (C) – (iv), (D) – (i)
(4) (A) – (iii), (B) – (i), (C) – (ii), (D) – (iv)

46. (2)
$TiCl_3$ = Ziegler – Natta polymerization
$PdCl_2$ = Wacker process
$CuCl_2$ = Deacon's process
V_2O_5 = Contact process

47. Which one has the highest boiling point?
(1) He (2) Ne (3) Kr (4) Xe

47. (4)
Xe has the highest boiling point.

48. The number of geometric isomers that can exist for square planar $[Pt (Cl) (py) (NH_3) (NH_2OH)]^+$ is (py = pyridine) :
(1) 2 (2) 3 (3) 4 (4) 6

48. (2)
No. of Geometrical isomers of $[Pt(Cl) (Py) (NH_3) (NH_2OH)]^+ = 3$

49. The color of $KMnO_4$ is due to :
(1) M $\rightarrow$ L charge transfer transition
(2) d – d transition
(3) L $\rightarrow$ M charge transfer transition
(4) $\sigma - \sigma^*$ transition

49. (3)

50. Assertion : Nitrogen and Oxygen are the main components in the atmosphere but these do not react to form oxides of nitrogen.

Reason : The reaction between nitrogen and oxygen requires high temperature.

(1) Both assertion and reason are correct, and the reason is the correct explanation for the assertion
(2) Both assertion and reason are correct, but the reason is not the correct explanation for the assertion
(3) The assertion is incorrect, but the reason is correct
(4) Both the assertion and reason are incorrect

50. (1)

51. In Carius method of estimation of halogens, 250 mg of an organic compound gave 141 mg of AgBr. The percentage of bromine in the compound is :
(*at. mass Ag* = 108; *Br* = 80)

(1) 24 (2) 36 (3) 48 (4) 60

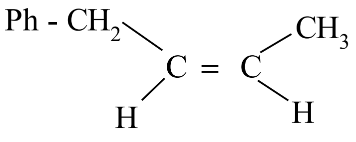
51. (1)

% of Bromine = $\frac{80}{188} \times \frac{141\,\text{mg}}{250\,\text{mg}} \times 100 = 24\ \%$

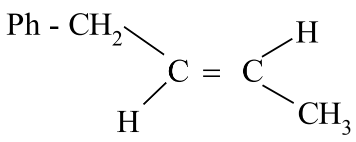
52. Which of the following compounds will exhibit geometrical isomerism?

(1) 1 – Phenyl – 2 – butene (2) 3 – Phenyl – 1– butene
(3) 2 – Phenyl – 1– butene (4) 1, 1 – Diphenyl – 1 – propane

52. (1)

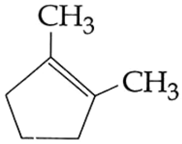


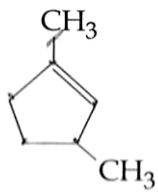
Cis

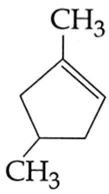


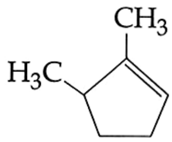
Trans

53. Which compound would give 5 – keto – 2 – methyl hexanal upon ozonolysis?

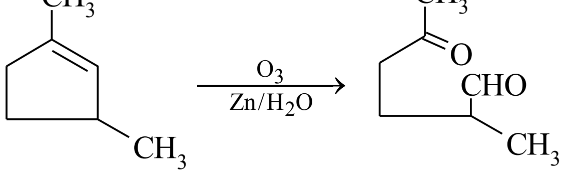
(1) 

(2) 

(3) 

(4) 

53. (2)



5 - keto - 2 - methyl hexanal

54. The synthesis of alkyl fluorides is best accomplished by :

(1) Free radical fluorination (2) Sandmeyer's reaction
(3) Finkelstein reaction (4) Swarts reaction

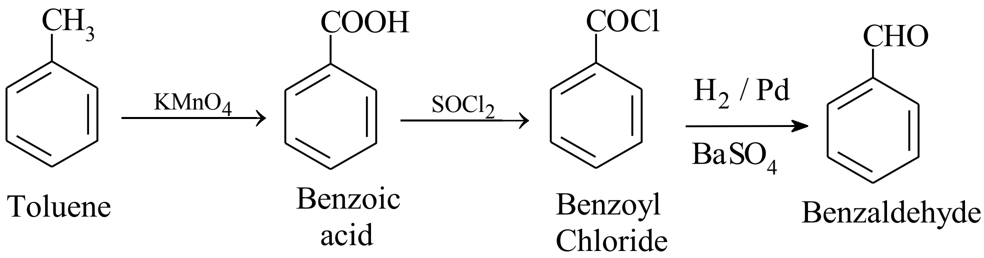
54. (4)

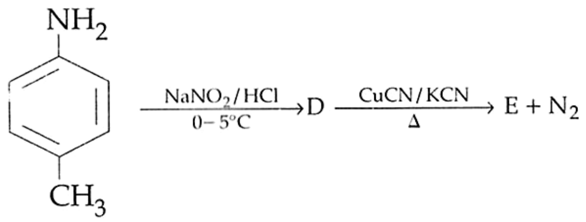
$R - I + AgF \longrightarrow R - F + AgI$ (Swarts Reaction)

55. In the following sequence of reactions : Toluene $\xrightarrow{KMnO_4}$ A $\xrightarrow{SOCl_2}$ B $\xrightarrow[BaSO_4]{H_2/Pd}$ C, the product C is :

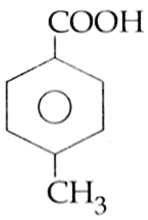
(1) C_6H_5COOH (2) $C_6H_5CH_3$ (3) $C_6H_5CH_2OH$ (4) C_6H_5CHO

55. (4)

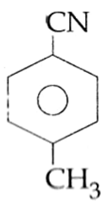


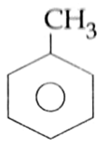
56. In the reaction 

the product E is :

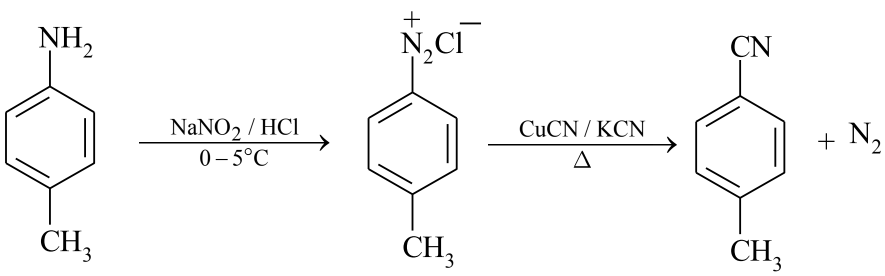
(1) 

(2) 

(3) 

(4) 

56. (3)



57. Which polymer is used in the manufacture of paints and lacquers?

(1) Bakelite (2) Glyptal (3) Polypropene (4) Poly vinyl chloride

57. (2)

Glyptal used in the manufacture of paints and lacquers.

58. Which of the vitamins given below is water soluble ?

(1) Vitamin C (2) Vitamin D (3) Vitamin E (4) Vitamin K

58. (1)

Vitamin 'B' and 'C' are water soluble.

59. Which of the following compounds is **not** an antacid?

(1) Aluminium hydroxide (2) Cimetidine

(3) Phenelzine (4) Ranitidine

59. (3)

Phenelzine is antidepressant drug.

60. Which of the following compounds is **not** colored yellow?

(1) $Zn_2[Fe(CN)_6]$ (2) $K_3[Co(NO_2)_6]$

(3) $(NH_4)_3\ [As\ (Mo_3\ O_{10})_4]$ (4) $BaCrO_4$

60. (1)

$Zn_2\ [Fe(CN)_6]$ is bluish white ppt.

PART- C : MATHEMATICS

61. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set A × B, each having at least three elements is :

(1) 219 (2) 256 (3) 275 (4) 510

61. (1)

Set A has 4 elements

Set B has 2 elements

∴ Number of elements in set (A × B) = 4 × 2 = 8

∴ Total number of subsets of (A × B) = 2^8 = 256

Number of subsets having 0 elements = 8_{C_0} = 1

Number of subsets having 1 element each = 8_{C_1} = 8

Number of subsets having 2 elements each = $8_{C_2} = \frac{8!}{2!6!} = \frac{8\times}{2} = 28$

∴ Number of subsets having at least 3 elements

= 256 – 1 – 8 – 28 = 256 – 37 = 219

62. A complex number z is said to be unimodular if $|z| =$. Suppose z_1 and z_2 are complex numbers such that $\frac{z\ -2z_2}{2-z_1 z}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a :

(1) straight line parallel to x–axis (2) straight line parallel to y–axis

(3) circle of radius 2 (4) circle of radius $\sqrt{2}$

62. (3)

$\left|\frac{z_1 - 2z}{2 - z_1\bar{z}}\right| = 1 \Rightarrow |z_1 - 2z_2|^2 = |2 - z_1\ \bar{z}_2|^2$

$\Rightarrow (z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}\ ) = (2 - z_1\ \bar{z}_2)(2 - \bar{z}_1\ z_2)$

$\Rightarrow z_1\ \bar{z}_1 + 4\ z_2\ \bar{z}_2 = 4 + z_1\ \bar{z}_1\ z_2\ \bar{z}_2$

$\Rightarrow 4 + |z_1|^2\ |z_2|^2 - 4\ |z_2|^2 - |z_1|^2 = 0$

$\Rightarrow (|z_2|^2 - 1)\cdot(|z_2|^2 - 4) = 0$

But $|z_2| \neq 1$, ∴ $|z_2| = 2$

Hence, z lies on a circle of radius 2 centered at origin.

63. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\dfrac{a_{10} - 2a_8}{2a_9}$ is equal to :

(1) 6 (2) −6 (3) 3 (4) −3

63. (3)

$x^2 - 6x - 2 = 0$

$\Rightarrow x^{10} - 6x^9 - 2x^8 = 0$

'α', β roots $\Rightarrow \alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0$ …(1)

& $\beta^{10} - 6\beta^9 - 2\beta^8 = 0$ …(2)

(1) – (2)

$\alpha^{10} - \beta^{10} - 6(\alpha^9 - \beta^9) - 2(\alpha^8 - \beta^8) = 0$

$\Rightarrow a_{10} - 6a_9 - 2a_8 = 0$

$\Rightarrow \dfrac{a_{10} - 2a_8}{2a_9} = 3$

64. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3 × 3 identity matrix, then the ordered pair (a, b) is equal to :

(1) (2, −1) (2) (−2, 1) (3) (2, 1) (4) (−2, −1)

64. (4)

$A\,A^T = 9I$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = 9I$$

$$\Rightarrow \begin{bmatrix} 9 & 0 & a+4+2b \\ 0 & 9 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b \end{bmatrix} \Rightarrow \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

Equation $a + 4 + 2b = 0 \Rightarrow a + 2b = -4$ …(1)

$2a + 2 - 2b = 0 \Rightarrow 2a - 2b = -2$ …(2)

& $a^2 + 4 + b^2 = 0 \Rightarrow a^2 + b^2 = 5$ …(3)

Solving $a = -2, b = -1$

65. The set of all values of λ for which the system of linear equations :

$2x_1 - 2x_2 + x_3 = \lambda x_1$

$2x_1 - 3x_2 + 2x_3 = \lambda x_2$

$-x_1 + 2x_2 = \lambda x_3$

has a non–trivial solution,

(1) is an empty set (2) is a singleton

(3) contains two elements (4) contains more than two elements

65. (3)

$(2 - \lambda)x_1 - 2x_2 + x_3 = 0$

$2x_1 - (3 + \lambda)x_2 + 2x_3 = 0$

$-x_1 + 2x_2 - \lambda x_3 = 0$

Non-trivial solution

$\Delta = 0$

$$\begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -3-\lambda & 2 \\ -1 & & -\lambda \end{vmatrix} = 0$$

$(2-\lambda)\ \{3\lambda+\lambda^2-4\}+2\cdot\{-2\lambda+2\}+(4-3-\lambda)=0$
$\Rightarrow(6\lambda+2\lambda^2-8-3\lambda^2-\lambda^3+4\lambda)-4\lambda+4+1-\lambda=0$
$\Rightarrow-\lambda^3-\lambda^2-5\lambda+3=0$
$x^3-\lambda^2+2\lambda^2-2\lambda-3\lambda+3=0$
$\lambda^2(\lambda-1)+2\lambda(\lambda-1)-3(\lambda-1)=0$
$(\lambda-1)(\lambda^2+2\lambda-3)=0$
$(\lambda-1)(\lambda+3)(\lambda-1)=0$
$\lambda=1, 1, -3$

66. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is :

(1) 216 (2) 192 (3) 120 (4) 72

66. (2)

Case – 1

Any 5 – digit number > 6000 is all 5-digits number

Total number > 6000 using 5 – digits = 5! = 120

Case – 2

Using 4 – digits

___	___	___	___
↓	↓	↓	↓
Can be 6, 7 or 8	4 ways	3 ways	2 ways

i.e. 3 ways

Total number = 3 × 4 × 3 × 2 = 72

Total ways = 120 + 72 = 192

67. The sum of coefficients of integral powers of x in the binomial expansion of $\left(1-2\sqrt{x}\right)^{50}$ is :

(1) $\frac{1}{2}\left(3^{50}+\ \right)$ (2) $\frac{1}{2}\left(3^{50}\right)$ (3) $\frac{1}{2}\left(3^{50}-\ \right)$ (4) $\frac{1}{2}\left(2^{50}+\ \right)$

67. (1)

$$\left(1-2\sqrt{x}\right)^{50}={}^{50}C_0-{}^{50}C_1\left(2\sqrt{x}\right)^1+{}^{50}C_2\left(2\sqrt{x}\right)^2-{}^{50}C_3\left(2\sqrt{x}\right)^3+{}^{50}C_4\left(2\sqrt{x}\right)^4\text{........}$$

So, sum of coefficient Integral powers of x

$$S={}^{50}C_0+{}^{50}C_2.2^2+{}^{50}C_4.2^4+\text{........}+{}^{50}C_{50}.2^{50}$$

Now, $\left(1+x\right)^{50}=1+{}^{50}C_1x+{}^{50}C_2x^2+{}^{50}C_3x^3+{}^{50}C_4x^4+\text{........}+{}^{50}C_{50}x^{50}$

Put x = 2, –2

$$3^{50}=1+{}^{50}C_1.2+{}^{50}C_2.2^2+{}^{50}C_3.2^3+{}^{50}C_4.2^4+\text{........}+{}^{50}C_{50}.2^{20} \qquad \text{........ (1)}$$

$$1=1-{}^{50}C_1.2+{}^{50}C_2.2^2-{}^{50}C_3.2^3+{}^{50}C_4 2^4-\text{........}+{}^{50}C_{50}.2^{50} \qquad \text{........ (2)}$$

(1) + (2)

$$3^{50}+1=2\left[1+{}^{50}C_2.2^2+{}^{50}C_4.2^4+\text{........}+{}^{50}C_{50}.2^{50}\right.$$

$$\therefore\ \frac{3^{50}+}{2}=1+{}^{50}C_2.2^2+{}^{50}C_4.2^4+....+{}^{50}C_{50}.2^{50}$$

68. If m is the A.M. of two distinct real numbers ℓ and n (ℓ, n > 1) and G1, G2 and G3 are three geometric means between ℓ and n, then $G_1^4+2G_2^4+G_3^4$ equals,

(1) $4\ell^2mn$ (2) $4\ell m^2n$ (3) $4\ell mn^2$ (4) $4\ell^2m^2n^2$

68. (2)

Given m is A.M. between ℓ & n

$\Rightarrow 2m = \ell + n$ …(1)

Given ℓ, G_1, G_2, G_3, n in G.P.

$r = \left(\frac{n}{\ell}\right)^{\frac{1}{4}} \Rightarrow r^4 = \frac{n}{\ell}$

$\therefore \; G_1 = \ell r,\ G_2 = \ell r^2,\ G_3 = \ell r^3$

So, $G_1^4 + 2G_2^4 + G_3^4 \quad = \ell^4 r^4 [1 + 2r^4 + r^8]$

$$= \ell^4 \cdot \left(\frac{n}{\ell}\right)\left[1 + \left(\frac{n}{\ell}\right) + \left(\frac{n}{\ell}\right)^2\right]$$

$$= n\ell^3 \left[\ \ + \frac{n}{\ell}\right]^2 = n\ell^3 \frac{(n+\ell)^2}{\ell^2}$$

$$= n\ell\ (2\,m)^2 = 4\ell\ m^2 n$$

69. The sum of first 9 terms of the series $\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + ...$ is

(1) 71 (2) 96 (3) 142 (4) 192

69. (2)

$$T_n = \frac{1^3 + 2^3 + ... + n^3}{1 + 3 + 5 + ... + (2n-1)} = \frac{\left\{\frac{n\,(n+1)}{2}\right\}^2}{\frac{n}{2}\left[1 + (2n-1)\right]} = \frac{(n+1)^2}{4}$$

$$\therefore \sum_{n=1}^{9} T_n = \frac{1}{4}\sum_{n=1}^{9}(n+1)^2 = \frac{1}{4}[1^2 + 2^2 + ... + 10^2 - 1^2]$$

$$= \frac{1}{4}\left[\frac{10(10+1)\,(2\times 10+1)}{6} - 1\right] = 96$$

70. $\lim\limits_{x\to 0} \frac{(1-\cos\ 2x)\ (3 \quad \cos\ x)}{x\ \tan\ 4x}$ is equal to :

(1) 4 (2) 3 (3) 2 (4) $\frac{1}{2}$

70. (3)

$$\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$$

$$\lim_{x\to 0}\frac{2\sin^2 x\,(3+\cos x)}{x\tan 4x}$$

$$\lim_{x\to 0}\frac{2\frac{\sin^2 x}{x^2}(3+\cos x)}{4\left(\frac{x\tan 4x}{4x^2}\right)} = 2$$

71. If the function, $g(x) = \begin{cases} k\sqrt{x+1} & ,\ 0 \le x \le 3 \\ mx + 2 & ,\ 3 < x \le 5 \end{cases}$ is differentiable, then the value of k + m is :

(1) 2 (2) $\frac{16}{5}$ (3) $\frac{10}{3}$ (4) 4

71. (1)

$$g(x) = \begin{cases} k\sqrt{x+1} & 0 \le x \le 3 \\ mx + 2 & 3 < x \le 5 \end{cases}$$

$$\lim_{x \quad 3} g(x) = \lim_{x \to 3^+} g(x) \quad g(3)$$

2k = 3m + 2 = 2k ……. (1)

$$g'(x) = \begin{cases} \dfrac{k}{2\sqrt{x+1}} & 0 \le \quad \le \\ m & 3 < x \le 5 \end{cases}$$

$$\underset{x=3}{\text{L.H.D at}} \quad \lim_{x \to 3^-} g'(x) = \frac{k}{4}$$

$$\underset{x=3}{\text{R.H.D at}} \quad \lim_{x \to 3^+} g'(x) = m$$

L.H.D. R.H.D.

$$\frac{k}{4} = m \qquad \text{……. (2)}$$

From (i) & (ii)

$$m \quad \frac{2}{5}, k = -$$

k + m = 2

72. The normal to the curve, $x^2 + 2xy - 3y^2 = 0$, at (1, 1) :

(1) does not meet the curve again

(2) meets the curve again in the second quadrant

(3) meets the curve again in the third quadrant

(4) meets the curve again in the fourth quadrant

72. (4)

$x^2 + 2xy - 3y^2 = 0$

$$2x + 2\left(x\frac{dy}{dx} + y\right) - 6y\frac{dy}{dx} = 0$$

$$2x + 2x\frac{dy}{dx} + 2y - 6y\frac{dy}{dx} = 0$$

$$(2x - 6y)\frac{dy}{dx} + (2x + 2y) = 0$$

$$\frac{dy}{dx} = \frac{-(x + \quad)}{(x - 3y)}$$

$$\left.\frac{dy}{dx}\right)_{(1,1)} = 1$$

Slope of normal is -1.

Equation of normal is $y - 1 = -1(x - 1)$

$x + y = 2$

$y = 2 - x$

$x^2 + 2x(2 - x) - 3(2 - x)^2 = 0$

$x^2 + 4x - 2x^2 - 3(4 + x^2 - 4x) = 0$

$x^2 + 4x - 2x^2 - 12 - 3x^2 + 12x = 0$

$-4x^2 + 16x - 12 = 0$

$x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0$

x = 1, 3

x = 1, y = 1

x = 3, y = −1

73. Let f(x) be a polynomial of degree four having extreme values at x = 1 and x = 2.

If $\lim\limits_{x\to 0}\left[1+\frac{f(x)}{x^2}\right] = 3$, then f(2) is equal to :

(1) – 8 (2) – 4 (3) 0 (4) 4

73. (3)

$$\lim_{x\to 0}\left(1+\frac{f(x)}{x^2}\right) = 3$$

$$\Rightarrow \lim_{x\to 0}\frac{f(x)}{x^2} = 2$$

$\therefore$ $f(x) = ax^4 + bx^3 + 2x^2 + 0x + 0$

$f'(x) = 4ax^3 + 3bx^2 + 4x$

$f'(1) = 4a + 3b + 4 = 0$ …. (1)

$f'(2) = 32a + 12b + 8 = 0$

$\Rightarrow 8a + 3b + 2 = 0$ …. (2)

Solving (1) and (2), we get $a = \frac{1}{2}$, $b = -2$

$\therefore$ $f(x) = \frac{x^4}{2} - 2x^3 + 2x^2$

$f(2) = 8 - 16 + 8 = 0$

74. The integral $\int \frac{dx}{x\ \left(x^4+1\right)^{3/4}}$ equals :

(1) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$ (2) $\left(x^4+1\right)^{\frac{1}{4}}\quad c$ (3) $-\left(x^4+1\right)^{\frac{1}{4}}+c$ (4) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}\quad c$

74. (4)

$$\int \frac{dx}{x\ .x^3\left(1+\frac{1}{x^4}\right)^{3/}}$$

$$= \int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/}}; \qquad \text{Let, } 1+\frac{1}{x^4} \qquad \Rightarrow \qquad -\frac{4}{x^5}dx \quad dt$$

$$= \frac{-1}{\quad}\int \frac{dt}{t^{3/}} = \frac{1}{4}\left|\frac{t^{\frac{-3}{4}+1}}{\frac{-3}{4}+1}\right. \quad c = \frac{-1}{4}\left[\frac{t^{1/}}{1/4}\right. + c$$

$$= -\ 1+\frac{1}{x^4}\Big)^{/4} + \quad = -\left(\frac{x^4+1}{x^4}\right.^{/4} \quad c$$

75. The integral $\int_2^4 \frac{\log x^2}{\log x \quad + \log\left(36-12x+x^2\right)}dx$ is equal to :

(1) 2 (2) 4 (3) 1 (4) 6

75. (3)

$$\int_2^4 \frac{\log x^2}{\log x^2 + \log(6-x)^2}dx$$

$$I = \int_2^4 \frac{\log x}{\log x + \log(6\quad x)}dx \qquad \text{........ (i)}$$

f (a + b – x) = f (x)

$$I = \int_2^4 \frac{\log(6-x)}{\log(6-x) + \log x}dx \qquad \text{........ (ii)}$$

(i) + (ii)

$$2I = \int_2^4 \frac{\log x + \log(6-x)}{\log(6-x) + \log x}dx$$

$$2I = \int_2^4 dx$$

2I = 4 – 2
2I = 2
I = 1

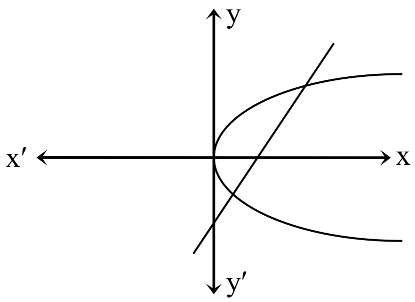
76. The area (in sq. units) of the region described by $\{(x, y) : y^2 \le 2x \text{ and } y \ge 4x - 1\}$ is :

(1) $\frac{7}{32}$ (2) $\frac{5}{64}$ (3) $\frac{15}{64}$ (4) $\frac{9}{32}$

76. (4)

$y^2 \le 2x$
& $y \ge 4x - 1$
By solving $y^2 = 2x$
& $y = 4x - 1$
$y = 1, \frac{-1}{2}$

$$A = \int_{\frac{-1}{2}}^{1} \left|\left(\frac{y+1}{4}\right) - \frac{y^2}{\ }\right| dy = \frac{9}{32}$$



77. Let y(x) be the solution of the differential equation $(x \log x)\frac{dy}{dx} + y = 2x \log x, (x \ge 1)$. Then y(e) is equal to :

(1) e (2) 0 (3) 2 (4) 2e

77. (3)

$$(x \ln x)\frac{dy}{dx} + y = 2x \log x, \qquad (x \ge 1)$$

put x = 1 then y = 0
Now equation can be written as

$$\frac{dy}{dx} + \frac{y}{x \log x} \quad 2$$

I.F. $= e^{\int \frac{1}{x \ln x} dx} = \ln x$

Solution of differential equation is

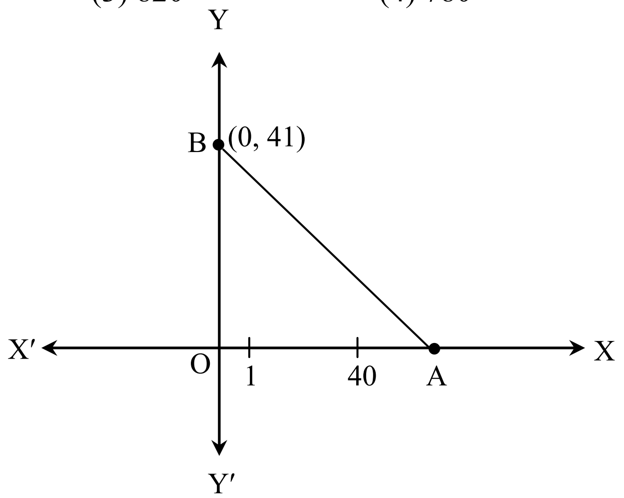
$$y.\ln x = c + \int 2.\ln x dx$$

y.lnx = c + 2 [x ln x – x]
Given at x = 1, y = 0

$0 = c + 2.(-1)$
$\Rightarrow c = 2$
$y.\ln x = 2 + 2[x\ln x - x]$
Put $x = e$
$y = 2 + 2[e - e]$
$y = 2$

78. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0), is :
(1) 901 (2) 861 (3) 820 (4) 780

78. (4)
$y - 0 = -1(x - 41)$
$x + y = 41$
$39 + 38 + 37 + ... + 1 = 39\frac{(39+1)}{2}$
$= 780$



79. Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, k ε R, is a :
(1) straight line parallel to x–axis. (2) straight line parallel to y–axis
(3) circle of radius $\sqrt{2}$ (4) circle of radius $\sqrt{3}$

79. (3)
$(2x - 3y + 4) + k(x - 2y + 3) = 0$ is family of lines passing through (1, 2). By congruency of triangles, we can prove that mirror image (h, k) and the point (2, 3) will be equidistant from (1, 2)
$\therefore$ (h, k) lies on a circle of radius = $\sqrt{(2-1)\ \ +(3-2)^2} = \sqrt{2}$

80. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is :
(1) 1 (2) 2 (3) 3 (4) 4

80. (3)
$x^2 + y^2 - 4x - 6y - 12 = 0$
$\therefore$ $C_1 = (2, 3)$ and $r_1 = \sqrt{4+9+12} = 5$
$x^2 + y^2 + 6x + 18y + 26 = 0$
$\therefore$ $C_2 = (-3, -9)$ and $r_2 = \sqrt{9+81-26} = 8$
$d(C_1, C_2) = \sqrt{(5)\ \ +(12)^2} = 13$
$|r_1 + r_2| = 8 + 5 = 13$
$\therefore$ $d(C_1, C_2) = r_1 + r_2$
$\therefore$ Number of common tangents = 3

81. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9} + \text{—} = 1$, is :
(1) $\frac{27}{4}$ (2) 18 (3) $\frac{27}{2}$ (4) 27

81. (4)

a = 3

$b = \sqrt{5}$

$\frac{\ ^{2}}{\ } = \frac{5}{3}$

$\therefore$ One of the end points of a latus rectum = $\left(2, \frac{5}{3}\right.$

$\therefore$ Equation of the tangent at $\left(2, \frac{5}{3}\right.$ is

$\frac{x \times 2}{9} + \frac{y}{3} \times \frac{5}{5} = 1 \Rightarrow \frac{x}{9/2} + \frac{y}{3} =$

Area of the rhombus formed by tangents = $\frac{1}{2} \times - \times 3 \times \quad = 27$ sq. units

82. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. It the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is :

(1) $x^2 = y$ (2) $y^2 = x$ (3) $y^2 = 2x$ (4) $x^2 = 2y$

82. (4)

Let Q = $(4t, 2t^2)$

and O = (0, 0)

$\therefore$ $P = \left(\frac{4t}{4} \quad \frac{2t^2}{\ }\right.$

$\therefore$ x = t,

$y = \frac{t^2}{2} \Rightarrow 2y = x^2$

83. The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane x − y + z = 16, is :

(1) $2\sqrt{14}$ (2) 8 (3) $3\sqrt{21}$ (4) 13

83. (4)

Let x = 3r + 2

y = 4r − 1

z = 12r + 2

$\therefore$ 3r + 2 − 4r + 1 + 12r + 2 = 16

$\Rightarrow$ r = 1

$\therefore$ (x, y, z) = (5, 3, 14)

Required distance = $\sqrt{4^2 + 3^2 + 12^2} = 13$

84. The equation of the plane containing the line 2x − 5y + z = 3; x + y + 4z = 5, and parallel to the plane, x + 3y + 6z = 1, is :

(1) 2x + 6y + 12z = 13 (2) x + 3y + 6z = −7

(3) x + 3y + 6z = 7 (4) 2x + 6y + 12z = −13

84. (3)

Put z = 0 in first two planes

$\therefore$ 2x − 5y = 3

and x + y = 5

$\Rightarrow$ x = 4, y = 1, when z = 0

Let x + 3y + 6z = k be a plane parallel to given plane.

$\therefore$ 4 + 3 + 0 = k $\Rightarrow$ k = 7

$\therefore$ x + 3y + 6z = 7 is required plane.

85. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non–zero vectors such that no two of them are collinear and $\left(\vec{a}\times\vec{b}\right)\times\vec{c}=\frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of sin θ is :

(1) $\frac{2\sqrt{\ }}{3}$ (2) $\frac{-\sqrt{2}}{3}$ (3) $\frac{2}{3}$ (4) $\frac{-2\sqrt{\ }}{3}$

85. (1)

$(\vec{a}\times\vec{b})\times\vec{c} = \frac{1}{3}|\vec{b}|\,|\vec{c}|\,\vec{a}$

$\Rightarrow\ -\left\{(\vec{c}\cdot\vec{b})\,\vec{a}-(\vec{c}\cdot\vec{a})\,\vec{b}\right\}=\frac{1}{3}|\vec{b}|\,|\vec{c}|\,\vec{a}$

$\Rightarrow\ (\vec{c}\cdot\vec{a})\,\vec{b}-(\vec{c}\cdot\vec{b})\,\vec{a} = \frac{1}{3}|\vec{b}|\,|\vec{c}|\,\vec{a}$

$\Rightarrow$ Equate, $-(\vec{c}\cdot\vec{b}) = \frac{1}{3}|\vec{b}|\,|\vec{c}|$

$\Rightarrow\ \cos\theta=-\frac{1}{3}$

$\therefore\ \sin\theta=\sqrt{1-\cos^2\theta} = \sqrt{1-\left(-\frac{1}{3}\right|^2} = \frac{2\sqrt{\ }}{3}$

86. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is :

(1) $\frac{55}{3}\left(\frac{2}{\ }\right)^{11}$ (2) $55\left(\frac{2}{3}\right)^{10}$ (3) $220\left(\frac{1}{3}\right)^{12}$ (4) $22\left(\frac{1}{3}\right)^{11}$

86. (1)

$n(S) = 3^{12}$

$n(E) = {}^{12}C_3\cdot 2^9$

$P(E) = \frac{n(E)}{n(S)} = \frac{{}^{12}C_3\cdot 2^9}{3^{12}} = \frac{220.2^9}{3^{12}} = \frac{55}{3}\left(\frac{2}{\ }\right)^{11}$

Note : According to the question, all given options are wrong.

87. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is :

(1) 16.8 (2) 16.0 (3) 15.8 (4) 14.0

87. (4)

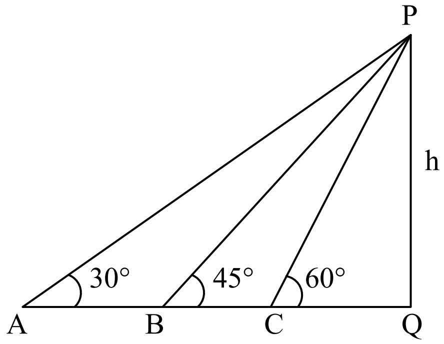
$\frac{\sum x_i}{16}=16\Rightarrow\sum x_i=256$

$\frac{\left(\sum x_i\right)-16+3+4+5}{18}=\frac{252}{18}\quad 14$

88. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30°, 45° and 60° respectively, then the ratio, AB : BC, is :

(1) $\sqrt{3}:1$ (2) $\sqrt{3}:\sqrt{2}$ (3) $1:\sqrt{3}$ (4) 2 : 3

88. (1)



$\frac{h}{AQ} = \frac{1}{\sqrt{3}}$

$\Rightarrow AQ = \sqrt{3}h$

Similarly, $BQ = h$

$CQ = \frac{h}{\sqrt{3}}$

$\therefore \quad \frac{AB}{BC} = \frac{AQ - BQ}{BQ - CQ} = \frac{(\sqrt{3}-1)h}{\left(h - \frac{h}{\sqrt{3}}\right)} = \frac{\sqrt{3}}{1}$

89. Let $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is :

(1) $\frac{3x - x^3}{1-3x^2}$ (2) $\frac{3x + x^3}{1-3x^2}$ (3) $\frac{3x - x^3}{1+3x^2}$ (4) $\frac{3x + x^3}{1+3x^2}$

89. (1)

$$\tan^{-1}(y) = \tan^{-1}\left(\frac{x + \frac{2x}{1-x^2}}{1 - \frac{x.2x}{1-x^2}}\right)$$

$$\Rightarrow \tan^{-1}(y) = \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right) \Rightarrow y = \frac{3x - x^3}{1-3x^2}$$

90. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to :

(1) $s \wedge \sim r$ (2) $s \wedge (r \wedge \sim s)$ (3) $s \vee (r \vee \sim s)$ (4) $s \wedge r$

90. (4)

$\sim [\sim s \vee (\sim r \wedge s)]$

$= \sim(\sim s) \wedge \sim (\sim r \wedge s)$

$= s \wedge (r \vee \sim s)$

$= (s \wedge r) \vee (s \wedge \sim s)$

$= s \wedge r \vee F$

$= s \wedge r$